
Exergy of solar radiation

Stephan Kabelac

University of the Federal Armed Forces,
Hamburg Institute of Thermodynamics, D-22039, Hamburg, Germany
E-mail: kabelac@hsu-hh.de

Abstract: Solar radiation reaching the ground is accompanied with radiation entropy. When the entropy production rate within any solar energy conversion device is to be calculated, the incoming radiation entropy flux has to be known. In this contribution first it is shown how the radiation entropy flux arriving on earth is to be calculated. Secondly, the interaction between the incoming radiation and the receiver surface is identified as one entropy production source. An approach for a reversible radiation conversion device is proposed. Maximum conversion efficiencies for non-concentrating solar energy converters are found to be between 50 – 77 % of the incoming radiation energy, depending on atmospheric conditions.

Keywords: solar energy; solar exergy; energy conversion; radiation entropy.

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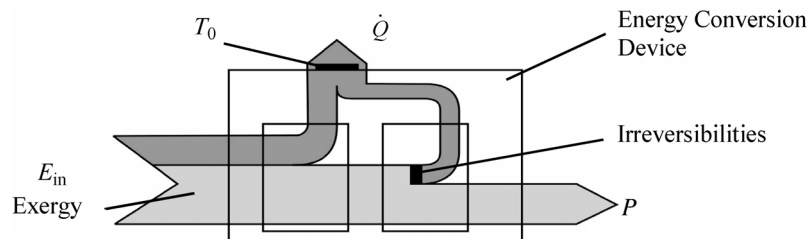
Biographical notes: Stephan Kabelac is Professor for Thermodynamics at the University of the Federal Armed Forces in Hamburg, Germany, since 2001. Before that he was the Chair of the Institute for Thermodynamics at the University of Hannover, Germany, since 1994. From 1991 to 1994 he was assigned to the distillation and heat transfer research group at the Bayer Chemical Company AG, in Leverkusen, Germany. His research interests are in evaporation heat transfer, thermodynamic evaluation of energy conversion devices such as fuel cells, and with the measurement of thermophysical properties such as speed of sound, optical properties and thermal conductivity.

1 Introduction

The conversion of solar energy into useful energy like mechanical or electrical energy, does not play an important role in the energy budget of most countries yet. But this energy conversion will become more important in future because of its environmentally perfect standing and it is important to have the thermodynamic tools ready for action when the demand increases. The problem with solar electromagnetic radiation is that it has an entropy content. Thus, if one wants to convert this entropy-accompanied form of energy into entropy-free mechanical or electrical power, one has to get rid of the incoming entropy, (Figure 1). A thermodynamic analysis of the very general conversion device shown in Figure 1, converting the incoming arbitrary energy flux E_{in} into mechanical or electrical power P , has to quantify the incoming exergy in a first step and then quantify the irreversibilities of the device itself in a second step. Given a fixed

environment, exergy is the fraction of the incoming energy, which is fully convertible into mechanical or electrical energy. Mechanical and electrical energy are completely exergy, they are fully convertible in all other energy types. Solar energy is not fully convertible because of its entropy content and thus its exergy content is less than 100%, so the situation shown in Figure 1 applies. Thus the energetic conversion efficiency of a solar conversion device will not be one, even if there were an ideal, fully reversible conversion. The exergy content of solar radiation reaching the surface of the earth is between 50% and 80% of its energy flux, depending on the atmospheric conditions, as will be shown below. So, if one aims to identify irreversible entropy production, i.e., exergy losses within a solar conversion device, one has to identify the entropy load, i.e., the exergy content, of the incoming solar radiation first. Only then one can quantify the entropy production rates within the device. A well-written review of some of the thermodynamic aspects of solar energy conversion is, for example, given by De Vos (1992). In the following, the procedure for the calculation of the exergy of solar radiation as a function of sun's position and atmospheric condition will be derived. First, the entropy content of incoming solar radiation will be examined by means of a model atmosphere.

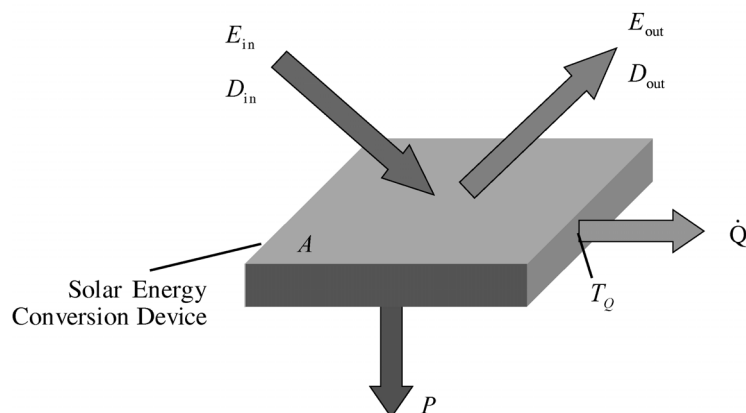
Figure 1 Energy and exergy fluxes in solar energy conversion device



2 The exergy of solar radiation

To find out the exergy content of incoming solar radiation, a simple solar energy conversion device is considered, shown in Figure 2:

Figure 2 A simple solar energy conversion device



This device is supposed to convert incoming solar radiation E_{in} into mechanical or electrical power P , i.e., pure exergy. The area open for radiation interference is A in m^2 . The energy balance equation for the device in Figure 2 in the case of steady state operation reads

$$P + \dot{Q} = A(E_{in} - E_{out}) \quad (1)$$

while the entropy balance equation is

$$\frac{\dot{Q}}{T_Q} = A(D_{in} - D_{out}) + \dot{S}_{irr}. \quad (2)$$

The radiation energy flux is denoted by E in W/m^2 , the accompanying radiation entropy flux by D in W/m^2K . The entropy production rate \dot{S}_{irr} in W/K is the overall production rate within the system boundary. Equations (1) and (2) are combined to give

$$P = A(E_{in} - E_{out}) - T_Q \cdot A(D_{in} - D_{out}) - T_Q \cdot \dot{S}_{irr}. \quad (3)$$

This equation tells us that for fixed incoming radiation the output power P of the conversion device will be large when the outgoing radiation energy flux E_{out} is small, the outgoing radiation entropy flux D_{out} is big, and the entropy production rate \dot{S}_{irr} is small. Under certain circumstances the power output P from the conversion device is equal to the exergy of the incoming radiation flux $Ex_{in} = f(E_{in}, D_{in}) = 0$. This is true when the entropy production rate \dot{S}_{irr} within the device is zero, i.e., no exergy is destroyed, when the heat is released into the environment at the environmental temperature T_0 and when the outgoing radiation flux do not carry away any exergy, i.e., $Ex_{out} = f(E_{out}, D_{out}) = 0$. If this were all true, Equation (3) would change to

$$Ex_{in} = P^{rev} = A(E_{in} - E_{out}) - T_0 \cdot A(D_{in} - D_{out}) \quad (4)$$

to give the exergy content. To express this exergy content of the incoming solar radiation, the entropy of the incoming and outgoing radiation fluxes have to be known. This will be addressed in the next step.

3 The entropy of radiation

The calculation procedure necessary to retrieve the energy and entropy radiation flux arriving on a horizontal surface on earth is shown in Figures 3 and 4. Several models have to be employed to incorporate the atmospheric conditions within the atmosphere, (see Figure 3). As the radiation entropy calculation procedure needs the spectral directional radiation intensity $L_\lambda = (\lambda, \Omega)$ and the spectral directional degree of polarisation $P_\lambda = P(\lambda, \Omega)$ in dependence of wavelength λ and solid angle Ω as input values (see Figure 4) (Kabelac and Drake, 1992). These values have to be calculated as a function of the position of the sun and atmospheric condition. The atmospheric model (Bird and Riordan, 1986) gives the direct and diffuse shortwave solar radiation energy and the long wave diffuse radiation emitted by the atmosphere itself. The solid angle distribution model (Rosen, Hooper and Brunger, 1989) calculates the solid angle of the direct solar radiation, the horizon brightening and the circumsolar diffuse radiation

distribution for the short wave diffuse radiation. The polarisation model (Coulson, 1988) shown in Figure 3 calculates the degree of polarisation of the shortwave diffuse solar radiation as a function of wavelength and solid angle.

Figure 3 The calculation procedure for the incoming solar radiation energy flux E_{in} and entropy flux D_{in}

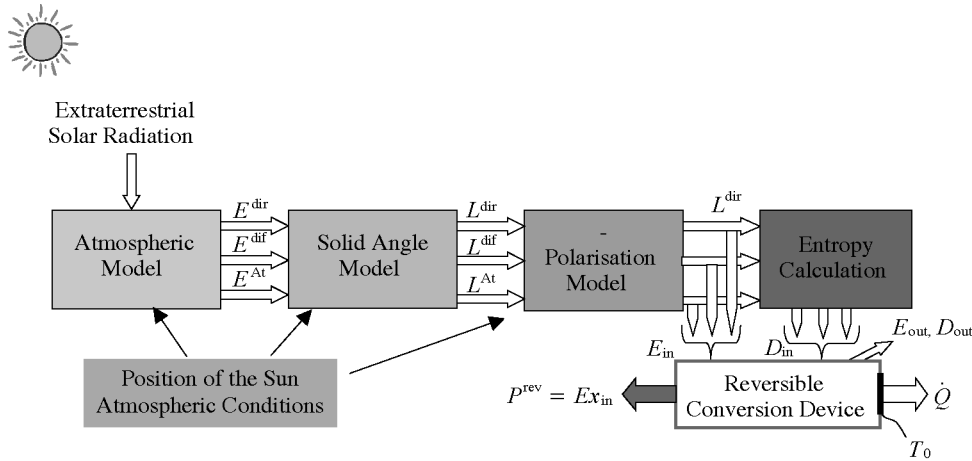


Figure 4 The calculation procedure for an arbitrary radiation energy and entropy flux

$$E = \int_{\lambda} E_{\lambda} d\lambda$$

$$E_{\lambda} = \int_{\omega} L_{\lambda} \cos \theta d\omega$$

$$L_{\lambda} = \frac{hc^2}{\lambda^5} x = L(\lambda, \omega) \begin{cases} L_{\lambda}^{max} = L_{\lambda} \cdot \frac{1+P}{2} \\ L_{\lambda}^{min} = L_{\lambda} \cdot \frac{1-P}{2} \end{cases}$$

Degree of Polarisation

$$P = \frac{L^{max} - L^{min}}{L^{max} + L^{min}} = P(\lambda, \omega)$$

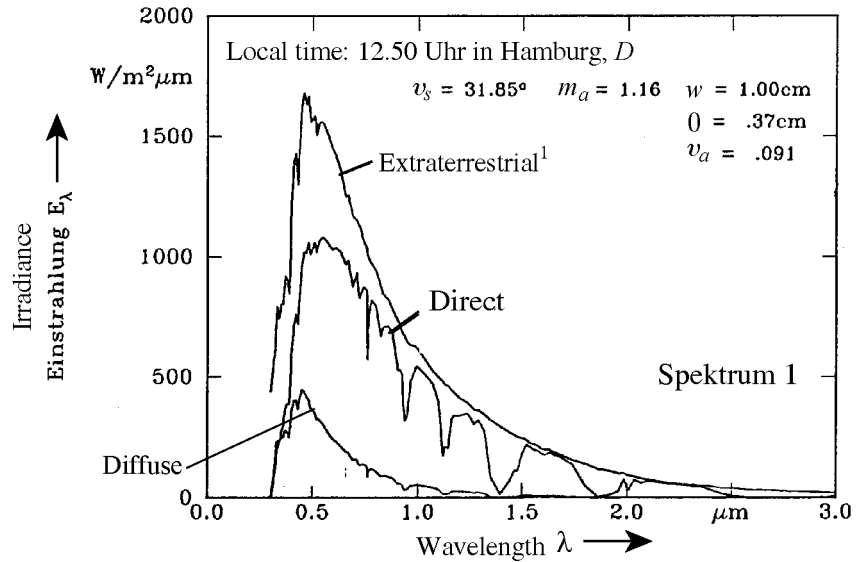
$$K_{\lambda} = \frac{kc}{\lambda^4} \left[\left(1 + \frac{\lambda^5 L_{\lambda}^{min}}{hc^2} \right) \ln \left(1 + \frac{\lambda^5 L_{\lambda}^{max}}{hc^2} \right) - \frac{\lambda^5 L_{\lambda}^{min}}{hc^2} \ln \left(\frac{\lambda^5 L_{\lambda}^{min}}{hc^2} \right) \right]$$

$$D = \int_{\lambda} \int_{\omega} \{K_{\lambda} (L_{\lambda}^{max}) + K_{\lambda} (L_{\lambda}^{min})\} \cos \theta d\omega d\lambda$$

The direct part of the solar radiation as well as the long wave atmospheric radiation are unpolarised. The radiation energy flux E_{in} arriving on the surface of the earth as an incoming radiation flux is found by integration of the spectral directional radiation energy

intensity L_λ over wavelength and solid angle (see Figure 4). A typical spectrum of short wave incoming spectral radiation energy E_λ as calculated with the model of Bird and Riordan (1986) is shown in Figure 5, valid for the city of Hamburg in northern Germany.

Figure 5 Typical spectra for noon on a summer day in Hamburg



Due to scattering in the atmosphere, the incoming diffuse short wave radiation is partly polarised. For an ideal Rayleigh-scattering atmosphere (Coulson, 1988) has published tables giving the degree of polarisation. A correlation fitted to this data is given in Kabelac (1994).

The entropy flux accompanying the incoming radiation energy flux is found from the radiation energy intensity and the degree of polarisation as was first shown by Planck (1923). With the procedure outlined in Figure 3 and Figure 4 the radiation energy flux E_{in} and the entropy flux D_{in} from Equation (4) are now known and are assumed to be fixed values for the energy conversion device.

The resulting values for the radiation entropy of incoming solar radiation can be approximately given as a function of the hemispherical radiation energy. The following correlations have been found by fitting a lot of calculation results for different atmospheric conditions in middle Europe.

The entropy content D_{dir} for the direct part of terrestrial solar radiation E_{dir} is approximately,

$$D_{dir} = C_{dir} \cdot E_{dir}^{0.9}$$

with E in W/m^2 , D in $W/m^2/K$ and the constant $C_{dir} = 0.000462$. For the diffuse part the entropy flux D_{dif} is calculated by

$$D_{dif} = C_{dif} \cdot E_{dif}^{0.9}$$

with $C_{dif} = 0.0014$.

These equations hold true for the short wave part of the incoming radiation. The atmosphere itself contributes its own long wave radiation ($3 < \lambda < 25 \mu\text{m}$) in addition to the short wave ($0.1 < \lambda < 3 \mu\text{m}$) solar radiation. This long wave radiation has a negligible effect on the energy balance equation in solar energy conversion, but unfortunately plays a role in the entropy balance equation (Kabelac, 1994). Based on data of Martin and Berdahl (1984) typical values of atmospheric long wave radiation entropy fluxes have been calculated to be $D_A = 0.757 \text{ W/m}^2\text{K}$ for a mean atmospheric temperature of $T_A = 273\text{K}$. This value has to be added to the short wave result for D_{in} .

4 Entropy production at the receiver surface

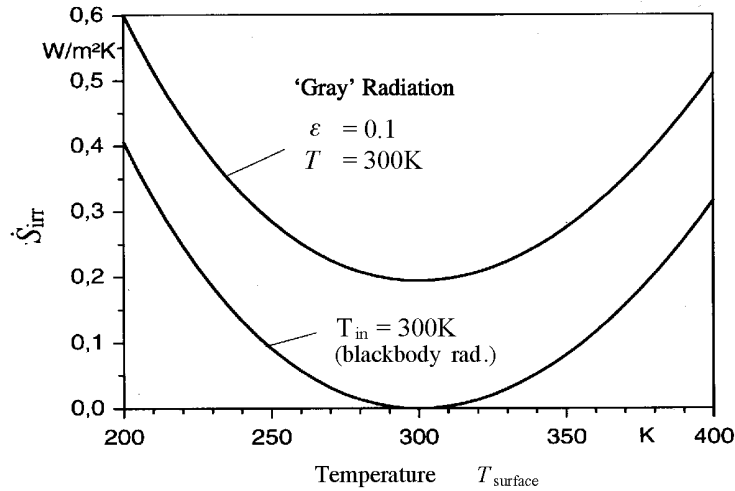
The equation giving the exergy of a radiation energy flux, Equation (4), also calls for the energy E_{out} and the entropy D_{out} of the outgoing radiation flux. This is troublesome, because for this reason some properties of the conversion device have to be fixed even though the exergy of radiation should be independent of any property of the conversion device. This is just a model helping within the calculation procedure. Any surface of a solar energy conversion device which is exposed to incoming radiation is emitting radiation on its own as well. This may be in part the reflected fraction from the incoming radiation, but in any way it is the emission of the surface itself due to its thermodynamic temperature T_{surface} . In deriving Equation (4) we demanded that the outgoing radiation flux E_{out} should be free of exergy. Taking a first guess, this could be true for blackbody radiation at environmental temperature T_0 . It seems impossible to gain any useful work from blackbody radiation at environmental conditions, so it should be free of exergy. To create such an outgoing radiation flux, the surface of the solar energy converter has to be a blackbody surface with the surface temperature T_0 , i.e., the environmental temperature. But a blackbody surface will, by definition, absorb all incoming radiation, and absorption of incoming radiation having a different temperature T_{in} than the absorbing surface temperature T_{surface} will necessarily create entropy:

$$\dot{S}_{\text{irr}} > 0 \quad \text{if} \quad T_{\text{in}} \neq T_{\text{surface}} = T_0$$

This was shown and proven by Max Planck (1923). So demanding the surface of the conversion device to be a blackbody surface at $T_{\text{surface}} = T_0$ will create entropy and thus be in contradiction with the assumption $\dot{S}_{\text{irr}} = 0$ leading to Equation (4). This is shown in Figure 6, where the entropy production rate during absorption of radiation is shown as a function of T . This phenomenon is very similar to the entropy production rate in heat transfer if there is a heat flux between two systems at different temperatures.

To enable a reversible radiation-surface interaction, the temperature of the surface has to be adapted to the temperature of the incoming radiation, T_{in} . Because only isotropic blackbody radiation is characterised by one single thermodynamic temperature, the arbitrary properties of the incoming solar radiation have to be characterised by a mean average thermodynamic temperature T_{in} calculated by

$$T_{\text{in}} = \frac{\partial E_{\text{in}}}{\partial D_{\text{in}}} = \lim_{E_{\text{out}} \rightarrow E_{\text{in}}} \left\{ \frac{E_{\text{in}} - E_{\text{out}}}{D_{\text{in}} - D_{\text{out}} + S_{\text{irr}}/A} \right\} = \frac{4}{3} \frac{E_{\text{in}}}{D_{\text{in}}}$$

Figure 6 The entropy production rate as a function of the blackbody surface temperature T_{surface} 

When the surface temperature T_{surface} of the device is made equal to this average temperature T_{in} , a reversible interaction between radiation and surface is possible without any entropy production. Unfortunately, the energy flux converted in the device will be infinitely small at reversible operation, which is the same problem as with a standard Carnot engine. Thus, the surface area of the reversible conversion device has to grow infinitely large. But this is a theoretical model only, having the sole purpose of calculating the exergy content of solar radiation arriving on earth. But this theoretical model reveals basic entropy production mechanisms within a solar energy conversion device. Some typical values for the exergy content $\zeta = 1 - T_o / T_{\text{surface}} = Ex_{\text{in}} / E_{\text{in}}$ for three different atmospheric conditions in Hamburg, Germany are given in Table 1:

Table 1 Exergy content ζ of solar radiation

	E_{in}	D_{in}	T_{surface}	ζ
clear atmosphere	1052 W/m ²	1,06 W/m ² K	1331 K	0,77
turbid atmosphere	415 W/m ²	0,904 W/m ² K	612 K	0,51
cloudy sky	447 W/m ²	0,98 W/m ² K	608 K	0,507

5 Conclusion

Simple thermodynamic evaluation of a solar energy conversion device shows a source of entropy production at the surface of the device where the interaction with the radiation takes place. The reason for this entropy production is a temperature mismatch between the incoming radiation temperature and the temperature of the surface of the device. This is similar to entropy production in heat transfer. The entropy production can approach zero if the receiver surface approaches the average thermodynamic temperature of the incoming radiation. Typical conversion efficiencies of an ideal reversible solar energy conversion device are between 50% and 80% depending on atmospheric conditions.

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